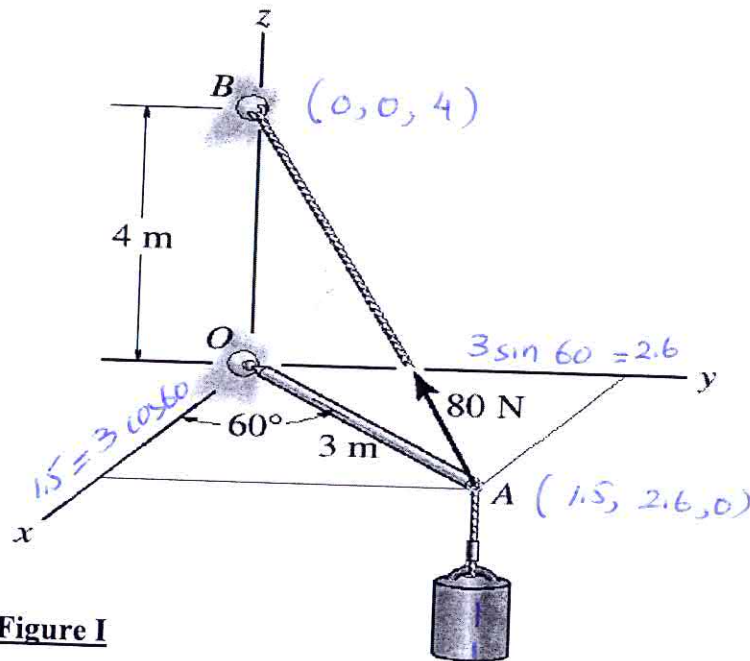


Problem I: (25 points)

Cable AB exerts a force of 80 N at the end of the 3-m long boom OA . The cylinder hung from point B weighs 150 N.

Determine the magnitude of the moment due to the cable AB force and the cylinder about point A using: (1) Cartesian approach and (2) Scalar approach.

Express this moment in Cartesian form and find its direction angles.

**Figure I**

$$\downarrow W = 150 \text{ N}$$

$$\vec{W} = -150\vec{k}$$

Calculations and/or Diagrams:

(1) Cartesian approach:

$$\vec{W} = -150\vec{k} \quad (1)$$

$$\vec{F}_{AB} = 80 \vec{u}_{AB}$$

$$\vec{u}_{AB} = \frac{-1.5\vec{i} - 2.6\vec{j} + 4\vec{k}}{\sqrt{1.5^2 + 2.6^2 + 4^2}} = \frac{-1.5\vec{i} - 2.6\vec{j} + 4\vec{k}}{5}$$

$$\vec{F}_{AB} = -24\vec{i} - 41.6\vec{j} + 64\vec{k} \quad (4)$$

$$\vec{M}_O = \vec{r}_{OA} \times \vec{F}_{AB} + \vec{r}_{OA} \times \vec{W}$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1.5 & 2.6 & 0 \\ -24 & -41.6 & 64 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1.5 & 2.6 & 0 \\ 0 & 0 & -150 \end{vmatrix} =$$

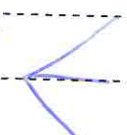
Calculations and/or Diagrams (cont'd):

$$= \vec{i}(2.6 \times 64) - \vec{j}(1.5 \times 64) + \vec{k}(1.5 \times -41.6 + 24 \times 2.6) \\ + \vec{i}(2.6 \times -150) - \vec{j}(1.5 \times -150) \quad (2)$$

$$= 166.4 \vec{i} - 96 \vec{j} + \vec{k}(-62.4 + 62.4) \\ \vec{M}_O = -223.6 \vec{i} + 129 \vec{j} - 390 \vec{i} + 225 \vec{j} \quad (2) \\ M_O = \sqrt{\quad} = 258.14$$

$$\alpha = \cos^{-1} \frac{-223.6}{258.14} = 150.01^\circ \\ \beta = \cos^{-1} \frac{129}{258.14} = 60^\circ \\ \gamma = \cos^{-1} 0 = 90^\circ \quad (2)$$

Scalar Approach

Due to $\vec{W} = -150\vec{k}$  $M_x = -150 \times 2.6$
 $M_y = +150 \times 1.5$

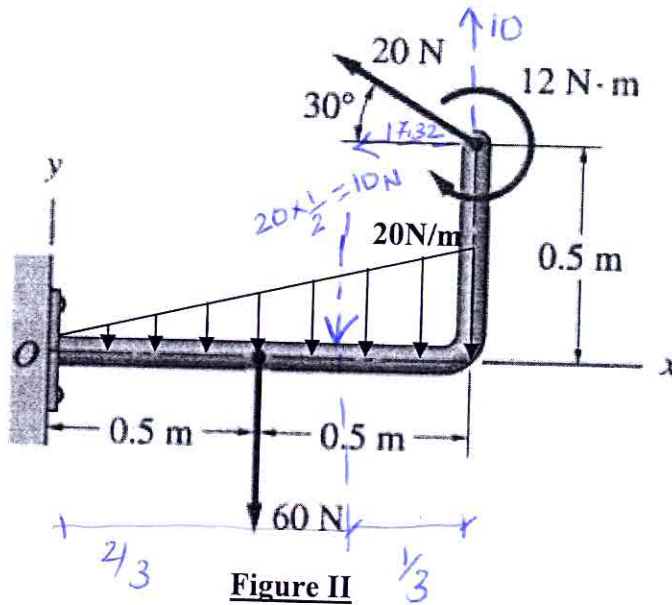
(10) Due to \vec{F}_{AB} $F_x = 24 \rightarrow 0$
 $F_y = -41.6 \rightarrow 0$
 $F_z = +64 \rightarrow M_x = +64 \times 2.6$
 $M_y = -64 \times 1.5$

$$\vec{M}_O = \vec{i}(-150 \times 2.6 + 64 \times 2.6) + \vec{j}(150 \times 1.5 - 64 \times 1.5) \\ = -223.6 \vec{i} + 129 \vec{j} \quad \checkmark$$

$$M_O = 258.14 \quad \checkmark$$

Problem II: (25 points)

Replace the force system acting on the frame by an equivalent resultant force and couple moment acting at point O. Can you replace the whole force system by a single Equivalent resultant force located on the x-axis? If yes, deduce the location of this resultant force on the x-axis.



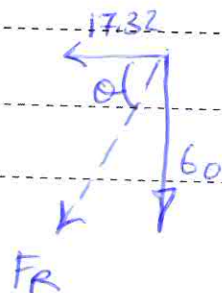
Calculations and/or Diagrams:

$$\vec{F}_R = ? = F_{Rx} \vec{i} + F_{Ry} \vec{j}$$

$$F_{Rx} = -20 \cos 30 = -17.32 \text{ N}$$

$$F_{Ry} = -60 - 10 + 20 \sin 30 = -60 \text{ N}$$

$$\vec{F}_R = -17.32 \vec{i} - 60 \vec{j} \quad F_R = \sqrt{\dots} = 62.45 \text{ N}$$

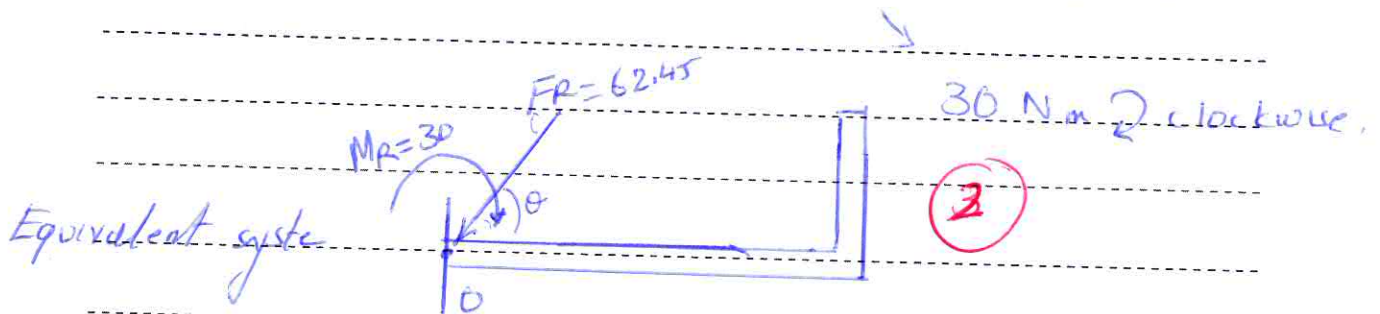


$$\tan \theta = \frac{60}{17.32} \rightarrow \theta = 73.9^\circ$$

Calculations and/or Diagrams (cont'd):

M_R about 0

$$\begin{aligned} \textcircled{9} \quad \sum M_{\text{at } 0} &= -12 + 10(1) + 17.32(0.5) \\ &\quad - 60(0.5) - 10\left(\frac{2}{3}\right) \\ &= -30 \text{ N}\cdot\text{m} \end{aligned}$$

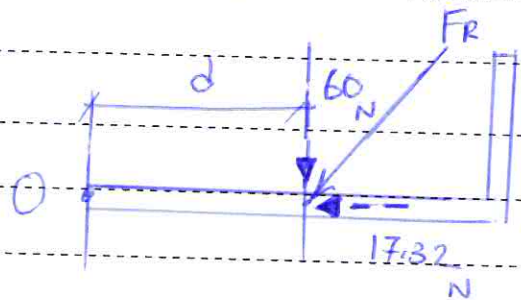


Can replace above system by a single force F_R located at d from '0'

$\textcircled{6}$

$$2 \cdot 60 \times d = 30$$

$$d = \frac{30}{60} = 0.5 \text{ m}$$



Problem III: (25 points)

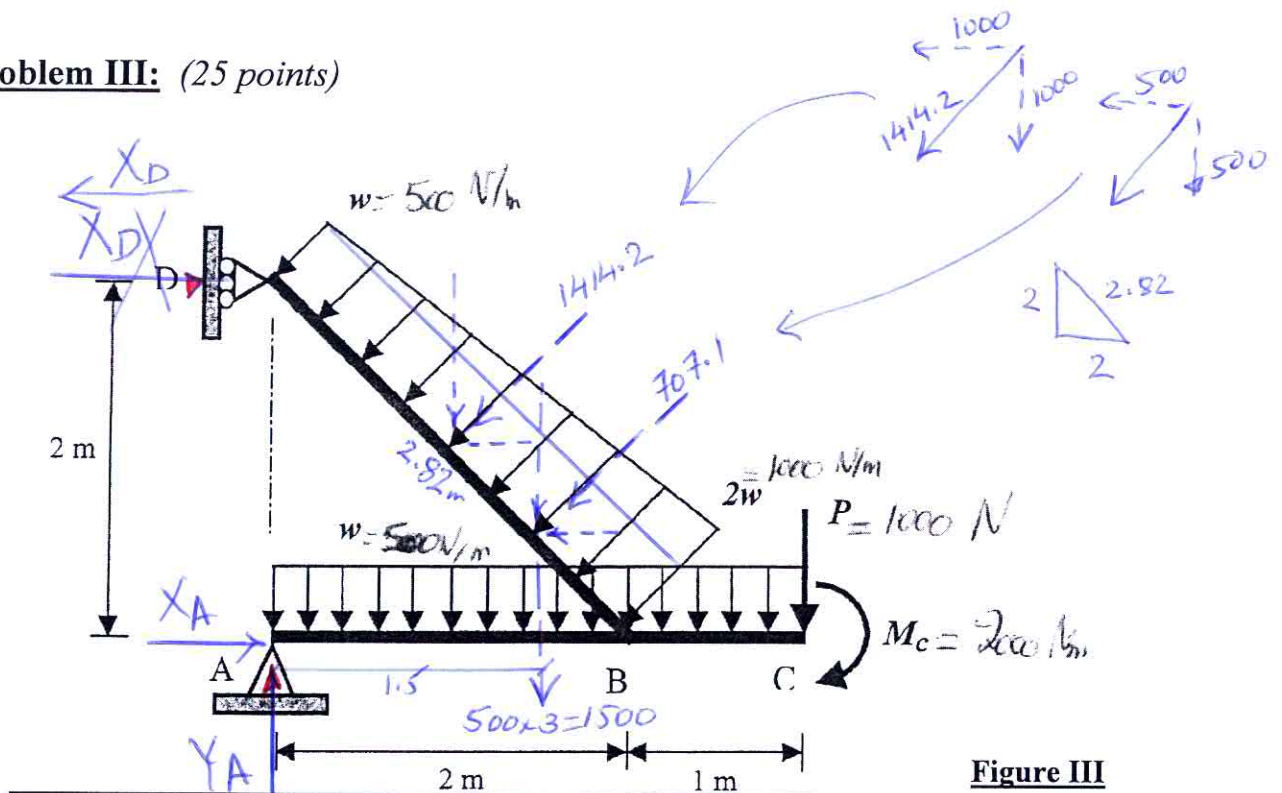


Figure III

The structural system shown in Figure III is stable and statically determinate.

Let $w=500 \text{ N/m}$, $P=1000 \text{ N}$, and $M_C=2000 \text{ Nm}$. Determine the reactions at supports A and D. and draw the updated FBD showing only the positive values and positive directions.

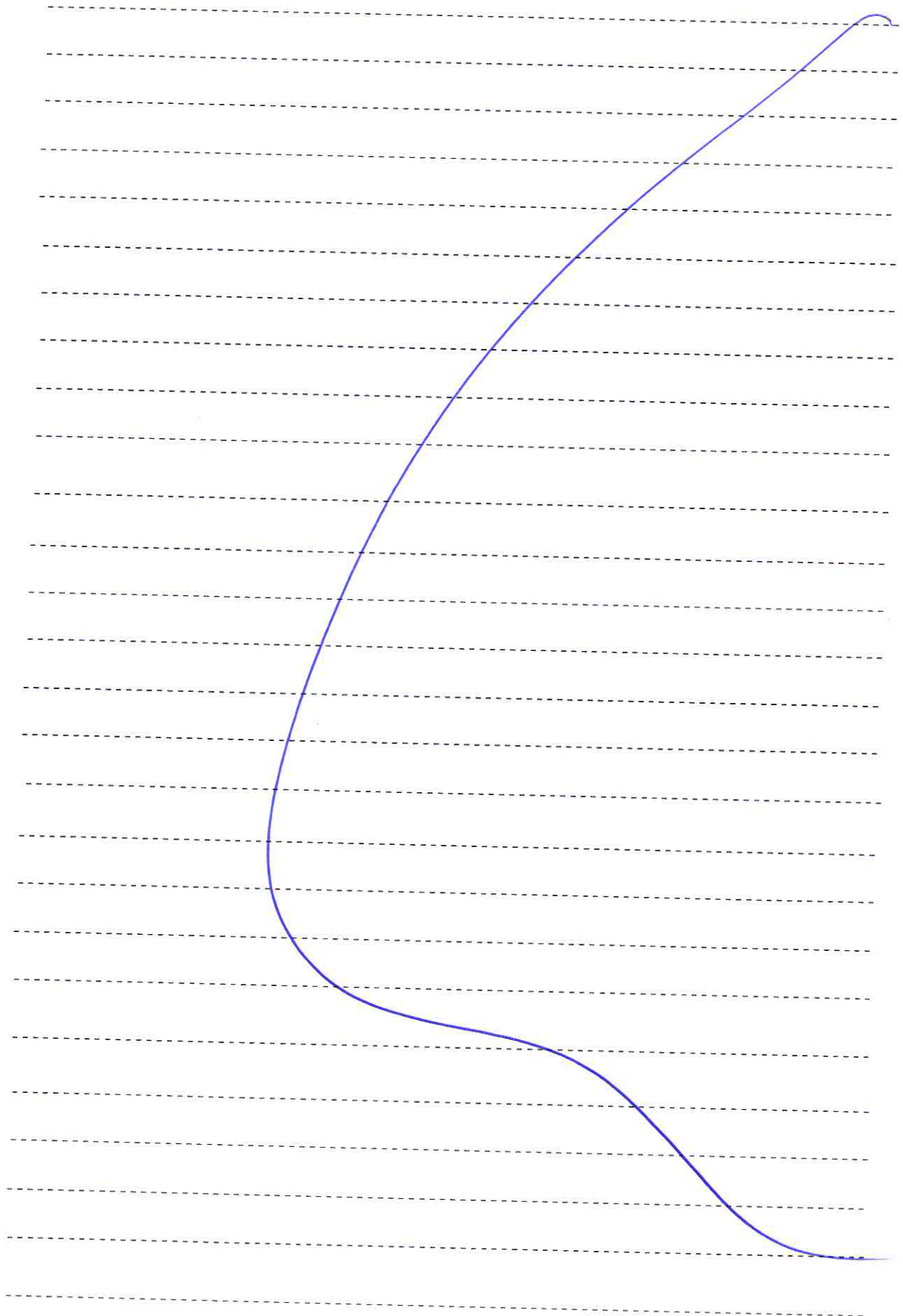
Calculations and/or Diagrams:

$$\begin{aligned} \sum M_D = 0 \\ -1414.2 \left(\frac{2.82}{2} \right) - 707.1 \times \frac{2.82 \times 2}{3} \\ - 1000(3) - 2000 - 1500(1.5) \\ + X_A (2) = 0 \\ X_A = 5286.66 \text{ N.} \end{aligned}$$

$$\begin{aligned} \sum F_x = 0 \quad X_A + X_D - 500 - 1000 = 0 \\ X_D = -3786.6 \text{ N.} \end{aligned}$$

$$\begin{aligned} \sum F_y = 0 \quad -1000 - 500 - 1000 - 1500 + Y_A = 0 \\ Y_A = 4000 \text{ N.} \end{aligned}$$

Calculations and/or Diagrams (cont'd):



Problem IV: (25 points)

The block W shown has a weight of 2500 N. Bar AB rests against a smooth vertical wall at end B and is supported at end A with a ball-and-socket joint. The two cables are attached to a point D on the bar midway between the ends. Determine the reactions at supports A and B and the tension in cable CD .

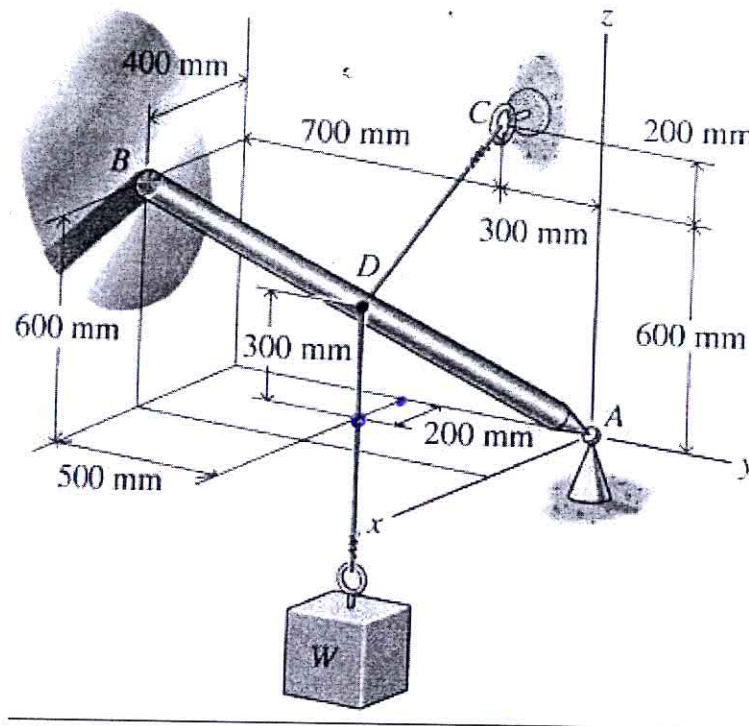


Figure IV

$A(0,0,0)$
 $B(400,-1000,600)$
 $C(0,-300,800)$
 $D(+200,-500,300)$

Calculations and/or Diagrams:

FBD

$\vec{Y}_B = Y_B \vec{j}$

$\vec{W} = -2500 \vec{k}$

$\vec{F}_{DC} = F_{DC} \vec{I}_{DC}$

$= F_{DC} (-200 \vec{i} + 200 \vec{j} + 500 \vec{k})$

$\sqrt{200^2 + 200^2 + 500^2} = 574.4$

Calculations and/or Diagrams (cont'd):

$$\vec{F}_D = -0.348 F_D \vec{i} + 0.348 F_D \vec{j} + 0.87 F_D \vec{k}$$

$$\downarrow \sum F_x = 0 \quad -0.348 F_D + A_x = 0 \quad (1)$$

$$\rightarrow \sum F_y = 0 \quad 0.348 F_D + A_y + Y_B = 0 \quad (2)$$

$$\uparrow \sum F_z = 0 \quad 0.87 F_D - 2500 + A_z = 0 \quad (3)$$

$$\curvearrowright \sum M @ x\text{-axis at } A = 0 = Y_B (600) + 2500(500) - (F_D \times 0.348)(300) - 0.87 F_D (500) = 0 \quad (4)$$

$$\rightarrow \sum M @ y\text{-axis at } A = 0 = -0.348 F_D (300) - 0.87 F_D (200) + 2500(200) = 0$$

$$F_D = \frac{2500 \times 200}{278.4}$$

$$F_D = 1796 \text{ N}$$

$$\text{From (3)} \quad A_z = 937.5 \text{ N}$$

$$\text{From (4)} \quad Y_B = 468.73 \text{ N}$$

$$\text{From (2)} \quad A_y = -1093.7 \text{ N}$$

$$\text{From (1)} \quad A_x = 625 \text{ N}$$

